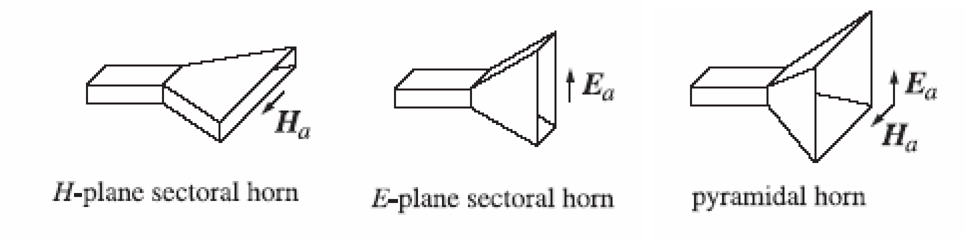
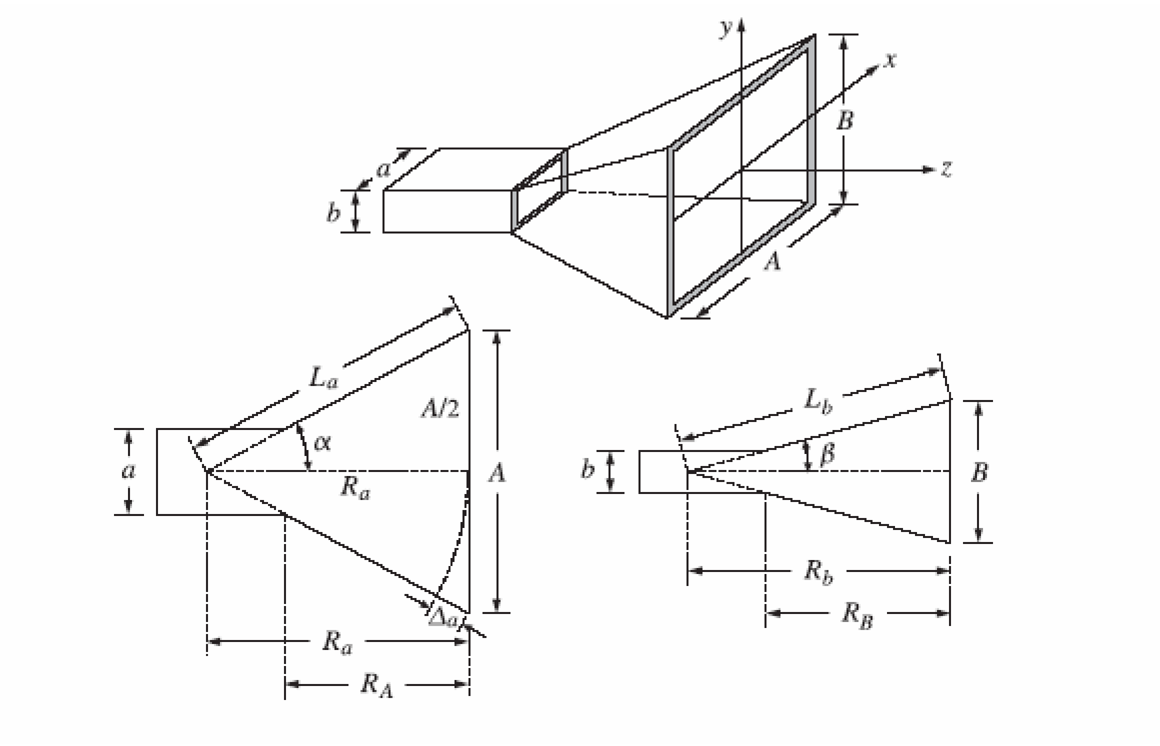
Horn antennas

The only practical way to increase the directivity of a waveguide is to flare out its ends into a horn. The most common types of horn are: the H-plane sectoral horn in which the long side of the waveguide (the a-side) is flared, the E-plane sectoral horn in which the short side is flared, and the pyramidal horn in which both sides are flared.



The pyramidal horn is the most widely used antenna for feeding large microwave dish antennas and for calibrating them. The sectoral horns may be considered as special limits of the pyramidal horn. In Figure is shown the geometry in more detail, moreover , the two lower figures are the crosssectional views along the xz- and yz-planes.



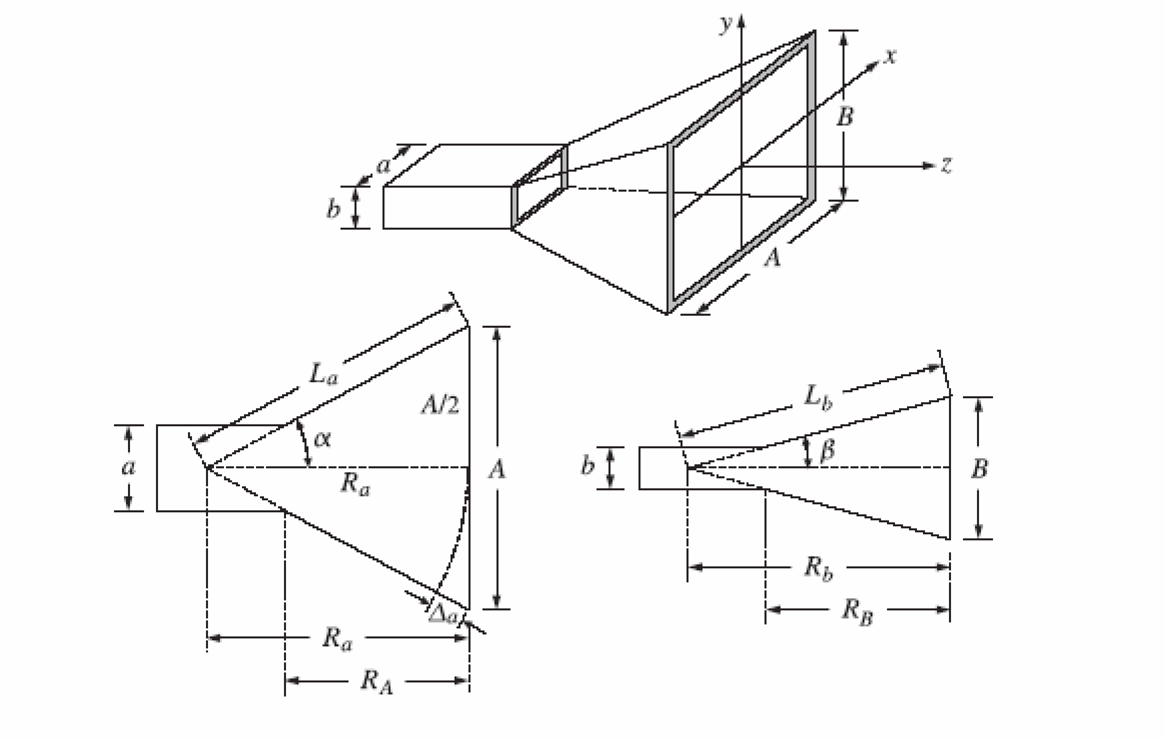
It follows from the geometry that the various lengths and flare angles are given by:

$$R_a = \frac{A}{A-a} R_A \qquad R_b = \frac{B}{B-b} R_B$$

$$L_a^2 = R_a^2 + \frac{A^2}{4} \quad L_b^2 = R_b^2 + \frac{B^2}{4}$$

$$\tan\alpha = \frac{A}{2R_a} \quad \tan\beta = \frac{B}{2R_b}$$

$$\Delta_a = \frac{A^2}{8R_a} \qquad \Delta_b = \frac{B^2}{8R_b}$$



Considering the geometry depicted in figures the the various lengths and flare angles are given by:

$$R_a = \frac{A}{A-a} R_A \qquad R_b = \frac{B}{B-b} R_B$$

$$L_a^2 = R_a^2 + \frac{A^2}{4} \quad L_b^2 = R_b^2 + \frac{B^2}{4}$$

$$\tan\alpha = \frac{A}{2R_a} \quad \tan\beta = \frac{B}{2R_b}$$

$$\Delta_a = \frac{A^2}{8R_a} \qquad \Delta_b = \frac{B^2}{8R_b}$$

The quantities R_A and R_B represent the perpendicular distances from the plane of the waveguide opening to the plane of the horn. Therefore, they must be equal, $R_A = R_B$.
Given the horn sides A,B and the common length R_A, it is possible to calculate all the relevant geometrical quantities required for the construction of the horn.
The lengths Δa and Δb represent the maximum deviation of the radial distance from the plane of the horn.
The expressions are obtained considering an approximation valid when $R_a \gg A$ and $R_b \gg B$.
The aperture electric field is assumed to have the following form

$$E_y(x, y) = E_0 \cos\left(\frac{\pi x}{A}\right) e^{-jk\frac{x^2}{2R_a}} e^{-jk\frac{y^2}{2R_b}} \qquad (1)$$

taking into account the relative phase differences at the point (x, y) on the aperture of the horn relative to the center of the aperture.
We note that at the connecting end of the waveguide the electric field is $E_y(x, y)=E_0 \cos(\pi x/a)$ and changes gradually into the form of Eq. (1) at the horn end. Because the aperture sides A,B are assumed to be large compared to λ, the Huygens source assumption is fairly accurate for the tangential aperture magnetic field, $H_x(x, y) = -\frac{E_y}{\eta}$ so that,

$$H_x(x, y) = -\frac{1}{\eta} E_0 \cos\left(\frac{\pi x}{A}\right) e^{-jk\frac{x^2}{2R_a}} e^{-jk\frac{y^2}{2R_b}}$$

The quantities $k\Delta_a$, $k\Delta_b$ are the maximum phase deviations in radians. Therefore, $\Delta a/\lambda$ and $\Delta b/\lambda$ will be the maximum deviations in cycles. We define:

$$S_a = \frac{\Delta_a}{\lambda} = \frac{A^2}{8\lambda R_a} \qquad S_b = \frac{\Delta_b}{\lambda} = \frac{B^2}{8\lambda R_b}$$

It turns out that the optimum values of these parameters that result into the highest directivity are approximately: $S_a = 3/8$ and $S_b = 1/4$. We will use these values later in the design of optimum horns. For the purpose of deriving convenient expressions for the radiation patterns of the horn, we define the related quantities:

$$\sigma_a = 4S_a = \frac{A^2}{2\lambda R_a} \qquad \sigma_b = 4S_b = \frac{B^2}{2\lambda R_b}$$

The near-optimum values of these constants are $\sigma_a = \sqrt{4S_a} = \sqrt{4(3/8)} = 1.2247$ and $\sigma_b = \sqrt{4S_b} = \sqrt{4(1/4)} = 1$. These are used very widely, but they are not quite the true optimum values, which are $\sigma_a = 1.2593$ and $\sigma_b = 1.0246$.

The design problem for a horn antenna is to determine the sides A,B that will achieve a given gain G and will also fit geometrically with a given waveguide of sides a, b, satisfying the condition $R_A = R_B$. The two design equations for A,B are then

$$G = e\frac{4\pi}{\lambda^2}AB \quad \frac{\sigma_a}{\sigma_b} = \frac{B(B-b)}{A(A-a)}$$

The design of the constant aspect ratio case is straightforward. Because $\sigma_b = r\sigma_a$, the second condition is already satisfied. Then, the first condition can be solved for A, from which one obtains B = rA and $R_A = A(A - a)/(2\lambda\sigma^2_a)$:

$$G = e\frac{4\pi}{\lambda^2}A(rA) \quad A = \lambda\sqrt{\frac{G}{4\pi er}}$$

where e is the aperture efficiency.